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1 About Mathematical Finance

1.1 An introduction to this unit

The unit covers the principle of hedging and pricing of derivatives by no arbitrage method in a binomial model. Although this model is interesting in its own right, and is often the paradigm of practice, here it is used primarily as a vehicle for introducing a simple setting the concepts needed for the continuous-time theory. The key probabilistic concepts of conditional expectation, martingales, transformation of measures and binomial representation theorem are presented in an accessible and simple framework in the discrete-time setting. The second part unit concentrates on classical Black-Scholes analysis, Itô’s lemma and simple arbitrage arguments are used to derive Black-Scholes partial differential equation for the fair value of an option. A variety of different types of options are considered and it is shown how, by suitably selecting boundary and terminal conditions for the Black-Scholes equation. Martingale Methods in derivative pricing is in introduced in a simple and accessible setting and include pricing of different types of derivatives using martingale approach. The unit conclude with a variety of ‘exotic options’.

1.2 Approach to teaching

1.3 Staff details

Unit Coordinator: Dr. Rehez Ahlip
Office Location: Room 50, Building ER, Parramatta Campus
Telephone: (02) 9685 9178
Email: r.ahlip@uws.edu.au

Teaching Staff: Dr. Rehez Ahlip
Office Location: Building: ER, Room No: 07 , Campus: Parramatta
Telephone: 9685 9178
Email: r.ahlip@uws.edu.au

1.4 Student consultation arrangements

Face to face: Monday 10.00am-11.00am, Tuesday 10am-12am.
Telephone: Mondays, Wednesdays.
Email: On any week day, lecturer will reply as soon as possible.
Students are also welcome to telephone at any time and leave a message and lecturer will return call as soon as possible.

1.5 Student feedback and improvements to the unit

The University values student feedback in order to improve the quality of its educational programs. As a result of student feedback, the following changes and improvements to this unit have recently been made: Compulsory attendance at lectures and tutorials and weekly assignments. Students have expressed that weekly assignments have forced them to keep up with work required. Also lectures and tutorial have been scheduled over two days rather than all on one day as students felt four hours in one day was too intense.

2 You and this unit

2.1 What is expected of you

Attendance
Students are expected to attend all lectures and tutorials throughout the semester. It has been observed that students that regularly miss lectures and tutorials make up the majority of failing students. Consequently each student is expected to attend at least 80% of scheduled classes in order to pass the unit, and failing to do so without reasonable explanation will result in an Absent Fail grade being awarded.

Students who miss lectures will need to read the appropriate sections in the textbook and/or obtain a copy of notes from a fellow student.

**Unit credit points and Workload**

This is a 10 credit point unit, so students are expected to work (on average) 10 hours per week. Given the fact that the class times for this unit totals 4 hours per week, in addition to attending all classes students are expected to work for 6 hours out of class in order to pass the unit.

This unit will require your full and continuous attention to maintain the highest possible grades. Some weeks you will spend more time on learning activities and assessments and in other weeks the workload will be somewhat less. It will be essential for you to keep up with the assigned core questions so that you are properly prepared for each assessment task.

There is no easy way to learn any mathematics, but an analogy with a sport like tennis holds; it looks easy when you see someone else do it, but doing it one's self requires hours of practise. Read the definitions, look at examples in the book and in class, and then try doing it yourself, and keep trying until you can do it alone. Talk with your fellow students about the problems. Not only because they may help you, but you may help them. One of the best ways to learn something is to try to explain it to someone else. Do tutorial problems, practice and then practice some more.

**Online learning**

Students should access vUWS and check their student email account at least twice a week.

**General conduct and behaviour**

According to the UWS Teaching and Learning code (http://policies.uws.edu.au/view.current.php?id=00139 ) you are required to:

- obtain the unit outline for this unit, by the end of the second teaching week;
- regularly and actively participate in all scheduled educational activities, which includes lectures, tutorial, laboratory sessions, online activities etc;
- give honest, helpful and courteous feedback to your lecturer(s)
- make every effort to undertake the work required to successfully complete this unit;
- submit work that is your own for any assessment task;
- not indulge in any behaviour that disrupts the teaching and learning environment, or negatively affects fellow students and university staff, and understand that the University will take action against such behaviour as outlined in the Misconduct - Students Non-Academic Misconduct Policy
- treat university property with due care and report and damaged or broken equipment.

In addition, you should:

- be on time to lectures, tutorial and laboratory sessions. If you are late, then please enter the lecture/tutorial room or lab with courtesy and consideration for others;
- pay attention in lectures, tutorials and laboratory sessions as this is where helpful information is given out of the assessment tasks;
- switch off your mobile phone
- ask questions about the content that you found difficult, immediately after the lecture, tutorial or lab session finishes. If this cannot be accomplished, then make sure you see your lecturer or tutor as soon as possible to resolve any problems.
2.2 What you can expect from me

Feedback
Your tutorial will provide you with oral and written feedback on class test papers in the tutorial during the next week following the class in which they were presented. You will also receive feedback after handing in your assignment questions.

Consultation
My consultation times are posted on my door and during these times I will be available to meet with you. If you are unable to see me during these times, please feel free to make an appointment that suits us both.

General conduct and behaviour
It is my aim to create a learning environment so that you may reach your full potential in this unit. Accordingly, you can expect from the lecturing staff in this unit to:

- prepare thoroughly for each teaching session;
- be on time for each lecture and tutorial session;
- ensure that you understand the unit requirements and material;
- be available to assist students during the consultation times (as indicated above in section 1.4);
- treat you equitably, and with courtesy and respect;
- report immediately, any issues or concerns related to student academic and non-academic misconduct to the relevant authority, according to the UWS Misconduct Policy.

Sometimes the best laid plans do go astray! In the unlikely occurrence of this happening, you will be notified about any changes to the scheduled activities, at least 24-hours in advance (if possible), via an announcement on vUWS

2.3 How to use this learning guide

This Learning Guide supplements the Unit Outline and is designed to help you navigate through the unit. It will help you focus on what you need to do to prepare for the various assessment tasks throughout the unit. You should consult the Learning Guide on a regular basis, as you plan your study, as this guide contains information on how best to prepare for each assessment task.

The Learning Guide also offers tips to assist you in developing the skills and techniques of an effective, independent learner.

However, if you have any particular problems or issues regarding this Unit, please take these up with the Unit Coordinator so that they may be resolved as soon as possible. As an adult learner, it is expected that you will be responsible for your own learning and take the necessary and appropriate steps to ensure your success.

2.4 Policy and how it affects you

The University has a number of policies that relate to teaching and learning. Important policies affecting students include

- Assessment Policy
- Examinations Policy
- Special Consideration Policy
- Review of Grade Policy
- Assessment Practice - Fundamental Code
- Misconduct - Student Academic Misconduct Policy (see extract of the policy below under the heading “What is Academic Misconduct?”)
• Misconduct - Student Non-academic Misconduct Policy
• Enrolment Policy (includes a section on the UWS Student Email Account)
• Bullying Prevention Policy and Guidelines
• Sexual Harassment Prevention Policy

There are two policies that relate to misconduct - academic and non-academic misconduct. Breaches of these policies can have very serious consequences. It is essential that you are familiar with these policies and how to avoid misconduct of any type.

2.4.1 What is Academic Misconduct?

Academic Misconduct may involve plagiarism, collusion or cheating. Plagiarism involves submitting or presenting work in a unit as if it were the student's own work when, in fact, it was not. Collusion includes inciting, assisting, facilitating, concealing or being involved in plagiarism, cheating or other academic misconduct with others. Cheating includes dishonest conduct (or attempted dishonest conduct) in exams. For the full definition of academic misconduct and the consequences of such behaviour, you are advised to read the Misconduct - Student Academic Misconduct Policy in its entirety, refer to: http://policies.uws.edu.au/view.current.php?id=00051

The School of Computing and Mathematics definitions of Minor and Substantial Breaches of the UWS Academic Misconduct policy now follow.

Plagiarism

Minor breach: A minor breach occurs when the weighting of the assessment task is 10% or less, and 20% or less of the work submitted is taken from another source without reference to the original source or author.

Substantial breach: A substantial breach occurs when:

1. The weighting of the assessment task is more than 10%, or 20% or more of the work submitted is taken from another source without reference to the original source.
2. If a student has been found to have already committed an act of plagiarism and warned about it, whether it be a minor or substantial breach, then the next allegation will be treated as a substantial breach.

Cheating

1. Dishonest or attempted dishonest conduct during an examination, for example speaking to other candidates or otherwise communicating with them, leaving answer papers exposed for other students to view and/or copy or attempting to view another student’s solutions, would be deemed as minor. However, if this behaviour continued after the student had been asked to desist, then the breach would be treated as substantial.
2. Bringing into the examination room any textbook, notebook, memorandum, other written material or mechanical or electronic device (including mobile phones), or any item not authorised by the examiner would be treated as minor. However, if the student does not surrender the unauthorised item, then a substantial breach would have occurred.
3. Writing an examination or part of it, or consulting any person or materials outside the confines of the examination room without permission to do so, would constitute a substantial breach.
4. Cheating in take-home examinations, which includes, but it not limited to: making notes, papers or answers in connection with the examination (in whatever form) to others without the permission of the relevant lecturer; receiving answers, notes or papers in connection with the examination (in whatever form) from another student, or another source without the permission of the relevant lecturer; and the unauthorised collaboration with another person or student in the formulation of an assessable component of work constitutes a substantial breach.

Other Academic Misconduct
1. Tampering or attempts to tamper with examination scripts, class work, grades or class records, will be regraded as substantial.

2. Failure to abide by the directions of an academic member of staff regarding the individuality of work to be handed in, will, in the first instance be treated as minor. However, any reoccurrence of such behaviour will be regarded as substantial.

3. Acquisition, attempted acquisition, possession or distribution of examination materials or information without the authorisation of the academic member of staff will be regarded as substantial.

4. Impersonation of another student in an examination or other class assignment will be regarded as substantial.

5. Falsification or fabrication of practical or laboratory reports will be regarded as substantial.

6. Non-authorised use of tape recording of lectures will be regarded as minor, except where the student/s has been asked to desist and refuses to comply. This continued abuse will be regarded as substantial.

There are many resources to help you avoid academic misconduct. The library staff can help you with referencing and the Student Learning Unit can assist with academic writing and plagiarism. If you are unsure about any of your work you should also ask your tutor or lecturer for advice and feedback.

2.4.2 What is Non-academic Misconduct?

Non-academic misconduct includes unlawful activities and crimes, falsifying documents (like a medical certificate or academic records), harassing other students (or staff), stealing or damaging university property (like library books or computers) and disrupting other students or staff. These are just some of the types of non-academic misconduct and while these things are rare they do happen. If you believe you have been the victim of non-academic misconduct or you are aware of any academic misconduct it is very important that you report it.

You should report all matters of academic misconduct directly to your Head of Program.

2.5 What to do if you have a problem/concern

If you have a concern about this unit please contact the unit coordinator in the first instance. If you would prefer to speak to someone else you are advised to contact your Head of Program (see the online handbook to identify your Head of Program and their contact details http://handbook.uws.edu.au/hbook/).

More information about resolving complaints is available on the UWS website. http://uws.clients.squis.net/opq/planning_and_quality/complaints_management_and_resolution

The University also has a confidential Complaints Handling department (see link above for contact details). You may contact this department of the University at any time however as we would appreciate the opportunity to resolve this directly first.

3 Teaching, Learning outcomes and assessment

On the successful completion of this unit, it is expected that the student will be able to:

1. Solve the fundamental problem of diffusions and use this result to construct solutions to diffusion equations with generalized boundary and initial values;

2. Solve Black-Scholes differential equation for the case of European calls and puts using the fundamental solution and apply solutions to price plain vanilla options;

3. Construct the Binomial model for stock options applied to derivatives;

4. Apply the concept of no arbitrage and its consequences to pricing of derivatives using the binomial model for stock options and construct hedging strategies;
5. Evaluate risk neutral probability and using these probabilities price European calls and put options within the binomial model. Price options by constructing replicating portfolios;

6. Apply stochastic calculus and the concept of no arbitrage to derive Black-Scholes partial differential equations satisfied by any derivative;

7. Calculate the prices of European calls and puts within the Black-Scholes framework;

8. Price European calls, puts and exotic options using martingale methods
Lectures

Lecture is three hours each week where information on each topic will be presented. Theory and examples will also be presented. Students are welcomed to ask questions as this often avoids misunderstanding. Lectures notes will sometimes be available from vUWS and students are expected to have read these notes at least once before the lecture. When no lecture notes are available, students should refer to appropriate section in the textbook. Students should also bring the recommended textbook to the lecture.

Tutorials

There is a one (1) hour tutorial class each week where students have the opportunity to consolidate their understanding of the previous week’s lecture material. Tutorial problems will be set each week and students will need to attempt to solve each of these problems before the tutorial class. This may require re-reading lecture notes or textbook, and/or looking at examples provided in lectures and textbook. Students are expected to work individually but discussion with fellow students is encouraged. By completing these tutorial questions you are preparing for class tests, assignments and the final examination. In the tutorial class students may seek further guidance on understanding the theory and/or have solution to problems fully explained. There will only be sufficient time to solve a few tutorial problems during the tutorial class, so preparation is essential if tutorials are going to be used effectively. The danger is time could be wasted going over problems that students would have been able to solve on their own if they had done the required preparation, leaving no time for more difficult problems. Students should be aware that in this unit, later topics build on the material covered earlier.

The following learning resources will be provided on the unit’s

Lecture notes: Lecture notes with the worked examples omitted will be posted before each lecture, so that you don’t have to copy down everything during the lectures and can focus on listening instead. You should print out these notes, bring them to the lecture, and fill in the examples during the lecture.

Tutorial exercises and worked solutions to these: A sheet with tutorial questions will be posted one week before each tutorial. You should work through these questions before the tutorial and in the tutorial raise any problems you may have had.

Complete worked solutions for all tutorial questions will be posted after each tutorial.

Doing the tutorial questions and checking your own work against the worked solutions is the best way of preparing for Class Tests and for the Final Exam.

Worked solutions to Class Tests: Complete worked solutions for Class Tests will be posted after each Class Test.

Sample Exam: A sample final exam paper will be posted before Stuvac. The main purpose of this paper is to give you a rough idea of the style of the questions which you should expect in the exam. The selection of sample questions should not be taken as a hint regarding the actual content of the exam. Do not base your exam preparation primarily on this (or any other) sample exam paper: Merely being able to reproduce solutions to sample exam questions will not be sufficient to pass the Final Exam. To prevent you from ignoring this advice, no solutions to this paper will be published. (Before you get upset about this: Remember that you’ll get worked solutions to all tutorial questions!)

A tentative schedule of topics and assessment tasks is given in the following table; covered learning outcomes are indicated. This schedule is correct at the time of this writing and may be changed if this becomes necessary during the semester. All material covered in lectures and tutorials is examinable, regardless of whether it is listed explicitly on this schedule or not. Visit http://platformweb.uws.edu.au for the timetable and for tutorial registration.
<table>
<thead>
<tr>
<th>Week</th>
<th>Dates</th>
<th>Topics</th>
<th>Text references</th>
<th>Assessments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>July 30 to August 3</td>
<td>Introduction: Definitions of European Calls and Puts, Forward price, Binomial Model. Lecture notes.</td>
<td>No tutorial</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>August 6 to August 10</td>
<td>The discrete random walk, Wiener Process, Option pricing using the binomial model.</td>
<td>Tutorials begin</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>August 13 to August 17</td>
<td>Derivation of the fundamental solution to the diffusion equation.</td>
<td>Application to specific boundary value problems. Lecture Notes.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>August 20 to August 23</td>
<td>Model for asset prices in continuous time. The Markov property. Itô’s formula,</td>
<td>Lecture Notes.</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>August 27 to August 31</td>
<td>Black-Scholes Formulae. Similarity solutions.</td>
<td>Lecture Notes.</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>September 3 to September 7</td>
<td>Black-Scholes model for asset price dynamics. Derivation of Black-Scholes-Merton differential equation.</td>
<td>Lecture Notes.</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>September 10 to September 14</td>
<td>Variations of Black-Scholes model: Options on dividend paying assets.</td>
<td>Lecture notes.</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>September 17 to September 21</td>
<td>Variations of Black-Scholes model: Options on dividend paying stock.</td>
<td>Lecture Notes.</td>
<td>Class Test 1, Lab Test 1 (covers Weeks 1-5)</td>
</tr>
<tr>
<td>9</td>
<td>September 24 to September 28</td>
<td>Mid Session Break</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>October 1 to October 5</td>
<td>Martingales, Martingale approach to option pricing</td>
<td>Lecture notes</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>October 8 to October 12</td>
<td>Martingales, Martingale approach to option pricing</td>
<td>Lecture notes</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>October 15 to October 19</td>
<td>Risk neutral measures, Forward measures, Market Price of Risk.</td>
<td>Lecture Notes</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>October 22 to October 26</td>
<td>Fixed income derivatives.</td>
<td>Lecture Notes</td>
<td>Class Test 2, Lab Test 2 (covers Weeks 6-11)</td>
</tr>
<tr>
<td>14</td>
<td>October 29 to November 2</td>
<td>REVISION</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The main references for this unit are:


Additional references are:

All of the above texts should be available from the [http://library.uws.edu.au](http://library.uws.edu.au) Parramatta.

Some web sites relevant to this unit are the following:

- [http://elearning.uws.edu.au](http://elearning.uws.edu.au) e-learning system
- [http://library.uws.edu.au](http://library.uws.edu.au) library
- [http://platformweb.uws.edu.au](http://platformweb.uws.edu.au) Timetable and tutorial registration

### 4 Assessment Criteria and Standards

#### 4.1 Standards

All assessment items are written tests consisting of one or several individual questions. The questions in a test are in general not of equal value, but the amount of marks available for a question reflects the length and difficulty of a complete solution. Each question will be marked separately and the total marks obtained for all questions will be added.

The assessment criteria for each question are the **correctness** and the **completeness** of the solution given by the student. **Marks will, in general, not be given for final answers, but for the argument leading to a solution**, so you must make sure that you show all working and fully explain and justify all steps of your solution.

A marking scheme, which is used consistently by all markers, indicates which steps attract marks. If a mistake is made in one of the steps of a solution, or if a step is missing or not sufficiently explained, an appropriate part of the marks for that step is deducted, depending on the severity of the mistake or the omission.

Examples of marking schemes are given below.

Your grade in an assessment item is determined by the percentage of the total marks for the assessment you achieve, according to the following key:

<table>
<thead>
<tr>
<th>Percentage of full marks</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>85% - 100%</td>
<td>H (High Distinction)</td>
</tr>
<tr>
<td>75% - 84%</td>
<td>D (Distinction)</td>
</tr>
<tr>
<td>65% - 74%</td>
<td>C (Credit)</td>
</tr>
<tr>
<td>50% - 64%</td>
<td>P (Pass)</td>
</tr>
<tr>
<td>0% - 49%</td>
<td>F (Fail)</td>
</tr>
</tbody>
</table>
Your final grade for the unit is determined by your total mark in the same way. Your total mark is the weighted sum of all your assessment marks with the weights specified in Section 4.2.

Note that, regardless of your total mark, you will receive an AF grade (Absent Fail) if you miss one of the mandatory assessment components without valid reason. The mandatory assessment components are: both Class Tests and the Final Exam.

### 4.2 Assessment Information

<table>
<thead>
<tr>
<th>Assessment</th>
<th>Weighting</th>
<th>Date of Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lab Test 1</td>
<td>10.0%</td>
<td>Week 8</td>
</tr>
<tr>
<td>Class Test 1</td>
<td>10.0%</td>
<td>Week 8</td>
</tr>
<tr>
<td>Lab Test 2</td>
<td>10.0%</td>
<td>Week 13</td>
</tr>
<tr>
<td>Class Test 2</td>
<td>10.0%</td>
<td>Week 13</td>
</tr>
<tr>
<td>Final Exam</td>
<td>60%</td>
<td>The Exam Period</td>
</tr>
</tbody>
</table>

**Class Tests:** There will be two Class Tests. See the Unit Schedule below for more details. The Class Test will be *closed book* written tests of duration 50 minutes.

No replacement test will be given. If special consideration is granted for missed tests, the weights will be transferred to the Final Exam.

**Computer Lab Tests:** There will be two Laboratory Tests. See the Unit Schedule below for more details. Duration 50 minutes.

**Final Exam:** The Final Exam is a 2 hour assessment, and covers all unit learning outcomes. It is a formal examination held during the Examination Period and is *closed book examination*, that is, only written implements and *non-programmable scientific calculators with no graphical display or symbolic computations* will be allowed.

**Assessment requirements:**

In order to pass this unit, you must satisfy all of the following conditions:

- Achieve an overall mark of at least 50
- Achieve a mark of at least 40% (24 out of 60) in the Final Exam.
- Attempt at least one Class Test.

A student who misses the Final Exam or Class Tests without making a successful application for Special Consideration will receive an AF grade (Absent Fail). A student whose Final Exam mark is below 40% will receive a CF grade (Compulsory Fail), regardless of their total mark.
Sample Questions for: Test 1, Test 2, and Final Examination

Question 1: European Calls and Puts
A European call and put option on the same security expire in three months, both have the same strike price of $20, and both sell for the price of $3. If the nominal continuously compounded interest rate is 19% and the stock price is currently $25.

(a) Is this consistent with absence of arbitrage?

(b) If your answer to (a) is that arbitrage is possible, how would you construct an arbitrage portfolio to take advantage of this situation?

**Marking Scheme**  
(a) Using Put-Call parity:

\[ C(S, t) = P(S, t) + S_t - Ke^{-r(T-t)} \]

1 mark → \[ C(S, t) = 3 + 25 - 20e^{-0.19(3)} \]
1 mark → \[ = 3 + 25 - 20e^{-0.57} \]
1 mark → \[ \approx 8.494 \]
1/2 mark → Hence since \( C(S, t) = 3 \) there is risk-less arbitrage

(b)

1/2 mark → Buy Call:  
1/2 mark → Write Put:  
1/2 mark → Short Stock:  
1/2 mark → Lend:  
1/2 mark → Proceeds:

\[ \frac{25 - 19.50}{20} \approx 0.494 \]

At Expiry

<table>
<thead>
<tr>
<th>( S(3) &lt; 20 )</th>
<th>( S(3) \geq 20 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long Call</td>
<td>( 0 )</td>
</tr>
<tr>
<td>Short Put</td>
<td>( -(20 - S(3)) )</td>
</tr>
<tr>
<td>Close short position on Stock</td>
<td>( -S(3) )</td>
</tr>
<tr>
<td>Risk-less Investment</td>
<td>( 20 )</td>
</tr>
<tr>
<td>Risk-less Investment</td>
<td>( 0 )</td>
</tr>
</tbody>
</table>
Question 2: Diffusion Equation

(a) Find similarity solution to the problem

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2} - \infty < x < \infty, \quad \tau > 0$$

with

$$u(x, 0) = H(x) = \begin{cases} 
1 & x \geq 0 \\
0 & x < 0 
\end{cases} \quad (1)$$

Hint: Use the substitution $u(x, \tau) = f(\xi)$ with $\xi = \frac{x}{\sqrt{\tau}}$. Further $f(-\infty) = 0$ and $f(\infty) = 1$.

(b) Show that $\frac{\partial u}{\partial x}$ is the fundamental solution $u_\delta(x, \tau)$, either by direct differentiation or by constructing the initial value problem that it satisfies.

Marking Scheme

1 mark $\rightarrow$ $u(x, \tau) = \frac{1}{2 \sqrt{\pi \tau}} \int_{-\infty}^{\infty} u(s, 0) e^{-\frac{(x-s)^2}{4\tau}} \, ds$

1 mark $\rightarrow$ letting: $z = \frac{(s-x)}{\sqrt{\tau}} \Rightarrow dz = \frac{1}{\sqrt{\tau}} \, ds$

3 mark $\rightarrow$ $u(x, \tau) = \frac{1}{2 \sqrt{\pi \tau}} \int_{-\infty}^{\infty} u(s, 0) e^{-\frac{(x-s)^2}{4\tau}} \, ds$

$= \frac{1}{2 \sqrt{\pi \tau}} \int_{-\infty}^{\infty} H(s) e^{-\frac{(x-s)^2}{4\tau}} \, ds$

$= \frac{1}{2 \sqrt{\pi}} \int_{-\infty}^{\infty} H(x - \sqrt{\tau}z) e^{-\frac{z^2}{4}} \, dz$

2 marks $\rightarrow$ But, $H(x - \sqrt{\tau}z) = \begin{cases} 
1 & z \leq \frac{x}{\sqrt{\tau}} \\
0 & z > \frac{x}{\sqrt{\tau}} 
\end{cases}$

3 mark $\rightarrow$ $u(x, \tau) = \frac{1}{2 \sqrt{\pi}} \int_{-\infty}^{x/\sqrt{\tau}} e^{-z^2/4} \, dz$

$= N \left( \frac{x}{\sqrt{\tau}} \right)$
**Question 3: Binomial Trees**  A stock price is currently $50. It is known that at the end of 6 months it will be either $60 or $42. The risk-free rate of interest with continuous compounding is 12% per annum. Calculate the value of a 6-month European call option on the stock with an exercise of $48. Verify that the no-arbitrage arguments and risk neutral arguments give the same answers.

**Marking Scheme**  At the end of six months the value of the option will be either $12 (if the stock price is $60) or $0 (if the stock price is $42). Consider a portfolio consisting of:

1 mark → $+\Delta : \text{shares}$
1 mark → $-1 : \text{option}$

The value of the portfolio is either $42\Delta$ or $60\Delta - 12$ in six months. If

1/2 mark → $42\Delta = 60\Delta - 12$
1/2 mark → i.e., $\Delta = 0.6667$
1 mark → the value of the portfolio is certain to be $28.$
1 mark → For this value of $\Delta$ the portfolio is therefore risk-less.
1 mark → The current value of the portfolio is: $(0.6667 \times 50 - f)$

1 mark → Since the portfolio must earn the risk-free rate of interest: $(0.6667 \times 50 - f) e^{0.12 \times 0.5} = 28$
1 mark → $f = 6.96$

where $f$ is the value of the option. The value of the option is therefore $6.96$

Risk Neutral Argument: Let $p$ be the probability of an upward stock price movement in a risk-neutral world. We must have

1 marks → $60p + 42(1 - p) = 50 \times e^{0.06}$
1 mark → i.e., $18p = 11.09$ or $p = 0.6161$

The expected value of the option in a risk-neutral world is:

1 marks → $12 \times 0.6161 + 0 \times 0.3839 = 7.3932$
1 mark → this has a present value of: $7.3932 e^{-0.06} = 6.96$
**Question 4: Black-Scholes Partial Differential Equation** Derive Black-Scholes partial differential equation satisfied by any derivative written on a stock whose dynamics are given by:

\[ dS(t) = \mu S(t)dt + \sigma dw(t) \]

**Marking Scheme** Let \( V(S, t) \) be the price at time \( t \) of a derivative contingent on \( S \). Then the random walk followed by \( V(S, t) \) is given by:

1 mark: \[ dV = \left( \mu S \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 S \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} \right) dt + \sigma S \frac{\partial V}{\partial S} dW(t) \]

1 marks: Consider the portfolio:
- \( 1 : \) derivative
- \( -\Delta : \) shares

The holder of this portfolio is long one derivative and short an amount \( \Delta \) of shares. Define \( \Pi \) as the value of the portfolio. The number \( -\Delta \) is as yet unspecified.

By definition:

1 mark: \( \Pi = V - \Delta S \) (2)

1 mark: The jump in the value of this portfolio in one-time step is: \( d\Pi = dV - \Delta dS \)

The random walk followed by \( \Pi \):

1 mark: \[ d\Pi = \left( \mu S \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 S \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} - \mu \Delta S \right) dt + \sigma S \frac{\partial V}{\partial S} dW(t) \]

1 mark: The random component can be eliminated by choosing: \( \Delta = \frac{\partial V}{\partial S} \) (3)

1 mark: This results in a portfolio whose increment is wholly deterministic: \( d\Pi = \left( \frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial S^2} \right) dt \)

Appealing to the concepts of arbitrage and supply and demand. The return on an amount \( \Pi \) invested in risk-less assets would see a growth of \( r\Pi dt \) in a time \( dt \).

Thus, we have

1 mark: \[ r\Pi dt = \left( \frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial S^2} \right) dt \] (4)

Substituting (1) and (2) into (3) and dividing throughout by \( dt \) we obtain

2 marks: \[ \frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0 \]
# 5 Unit Information

<table>
<thead>
<tr>
<th>Student Consultation</th>
<th>Current consultation times will be advised by lecturers in the first lecture and on vUWS. Students are advised to check vUWS regularly for updates. At time of printing Learning Guide, consultation times listed above. Please note that students are required to use their UWS account when emailing lecturers; emails sent from non-UWS accounts may fall victim to spam filters and may not be read/answered.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode of Delivery and Teaching Schedule</td>
<td>The unit consists of three hours of lectures and one hour of tutorials per week. Tutorials start in Week 2. <strong>Students can only attend a tutorial group for which they are registered.</strong> Tutorial places are <strong>allocated via Platform Web only.</strong> Visit <a href="http://platformweb.uws.edu.au">http://platformweb.uws.edu.au</a> for the timetable and tutor registration. Announcements and assessments will be posted and tutorial questions will be made available prior to tutorials on vUWS; visit <a href="http://elearning.uws.edu.au">http://elearning.uws.edu.au</a>.</td>
</tr>
<tr>
<td>Attendance Requirements</td>
<td>Attendance at lectures is not compulsory but <strong>highly recommended.</strong> Be advised that students not attending the lectures regularly account for a large proportion of those failing the unit! Attendance at tutorials is compulsory; quizzes at start of tutorials is an assessment component for the unit. Students who miss a tutorial quiz for legitimate reasons, e.g. illness, should contact their campus lecturer and/or tutor and provide appropriate documentation.</td>
</tr>
<tr>
<td>Essential Equipment and/or Resources</td>
<td>A non-programmable calculator</td>
</tr>
<tr>
<td>Student Workload</td>
<td>A 10 credit point unit has a workload of 10 hours per week. As class time for this unit totals 4 hours per week, a student attending all classes should expect to have to work for 6 hours out of class to pass the unit.</td>
</tr>
</tbody>
</table>
| Important Information, Policies and Procedures for Students | Students are responsible for familiarising themselves with current UWS policies. You can find the full details of all policies that apply to you as a UWS student at [http://www.uws.edu.au/policies/a-z](http://www.uws.edu.au/policies/a-z). Especially important for your studies in this unit are the following policies:  
  - Assessment and Examinations Policy  
  - Assessment Practice – Fundamental Code  
  - Misconduct – Student Academic Misconduct Policy (see extract of the policy below under the heading "What is Academic Misconduct?")  
  - Misconduct – Student Non-academic Misconduct Policy  
  - Enrolment Policy (includes a section on the UWS Student Email Account) |
6 Assessment Criteria and Standards

6.1 Standards

All assessments consist of several independent parts for which marks are allocated according to the assessment criteria (see below). Your grade is determined by the percentage of the total marks for the assessment that you achieve, according to the following key.

<table>
<thead>
<tr>
<th>Percentage of full marks</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>85% - 100%</td>
<td>H (High Distinction)</td>
</tr>
<tr>
<td>75% - 84%</td>
<td>D (Distinction)</td>
</tr>
<tr>
<td>65% - 74%</td>
<td>C (Credit)</td>
</tr>
<tr>
<td>50% - 64%</td>
<td>P (Pass)</td>
</tr>
<tr>
<td>0% - 49%</td>
<td>F (Fail)</td>
</tr>
</tbody>
</table>

Your final grade for the unit is determined by your total mark in the same way. Your total mark is the weighted sum of all your assessment marks with the weights specified in Section 4.2.

Note that, regardless of your total mark, you will receive an AF grade (Absent Fail) if you miss one of the mandatory assessment components without valid reason. The mandatory assessment components are: both Class Tests and the Final Exam.

6.2 Tutorials

Acquiring mathematical skills requires practice. It is not possible to learn how to solve a counting problem or how to write a proof purely by reading the textbook or watching the lecturer do it in class. Consequently, tutorial exercises are a crucial learning component of the unit.

Tutorial exercises, reinforcing the learning outcomes of the unit, are published weekly. You will work on these problems during the tutorial, but you should read through the questions beforehand, and make sure you have revised all the relevant sections of the lecture notes so that you are ready to start working on the problems as soon as the tutorial begins. During the tutorial you will work through the questions, alone or with one or two other people. The tutor will be able to help when you are stuck, and may sometimes explain a question or two on the board.

Class test and exam questions are similar in nature and difficulty to the tutorial questions. The tutorial quizzes are also based on the tutorial questions, but generally involve only the easiest and shortest questions. The best way to prepare for all of these is not to read the textbook or your lecture notes, but to do the tutorial questions. If you can do all the tutorial questions you will be able to do very well in the class tests and exams.